Solving Heating Problems with Phase Changes

The trick on mixing various hot and cold components of a system is to remember that ALL the processes must balance out completely. All the hot components must cool down (lose joules) by exactly the amount that the cold components heat up (gain joule). This can be written algebraically:

$$\text{process1} \; + \; \text{process2} \; + \; \text{process3} \; + \; \text{etc...} \; = 0$$

Where each process is an equation for heat representing one of the segments on the heating diagram which is show below. Let's assume this diagram is for water. Segment A to B is for heating ice to 0°C, B to C for melting ice to water at 0°C, C to D for heating water from 0°C to 100°C, and so on...
This ultimately leads to a rather LARGE algebraic equation with lots to keep track of, especially the signs on temperatures. Not to mention you have to know where you will end up on the diagram to truly set up the processes correctly.

An easier way is to realize that the answer is path independent and I can imagine it happening anyway I see fit as long as a quantify all steps. So I prefer to pull ALL COMPONENTS of the problem (whether ice, water, or steam) to ONE specific point on the diagram shown below. My personal favorite is point C. That is water at 0°C. Another words, calculate the total heat it takes to get each component to that point and write down (save to memory locations in your calculator) the joules it took to get there. Remembering that the colder components are heat sinks (joules drain to the cold side) count all those totals as negative values of heat (joules). Note: yes, the value of q would be + for such processes, however, I view this as a price to be paid, meaning I *owe* the cold side this amount of heat. The hot components are heat sources and you can associate a positive sign on those segments. Now total ALL of your processes (the positives and the negatives) together to get to point C.

At this point, you will either have heat left over (+joules) or a heat deficit (-joules). I seriously doubt that the joules will cancel perfectly and you get zero. Plus ALL your water is now liquid at 0°C (add all the masses). Now you can easily make one more calculation as to whether the left over amount of heat will heat the water up (+joules) to some temperature via $q = m\, C_s\, \Delta T$, or freezes the water (-joules) to make some ice via $q = m\, \Delta H_{\text{fusion}}$.

Now if you picked the wrong point to converge on, you might have to go through a couple more calculations. For example, you have so much heat left over that you heat the water all the way back up to the boiling point and beyond. This is doubtful though if you have a little common sense.