

CH301 Unit 1

GAS LAWS REVIEW: MODELING GASES, GAS LAW STOICHIOMETRY

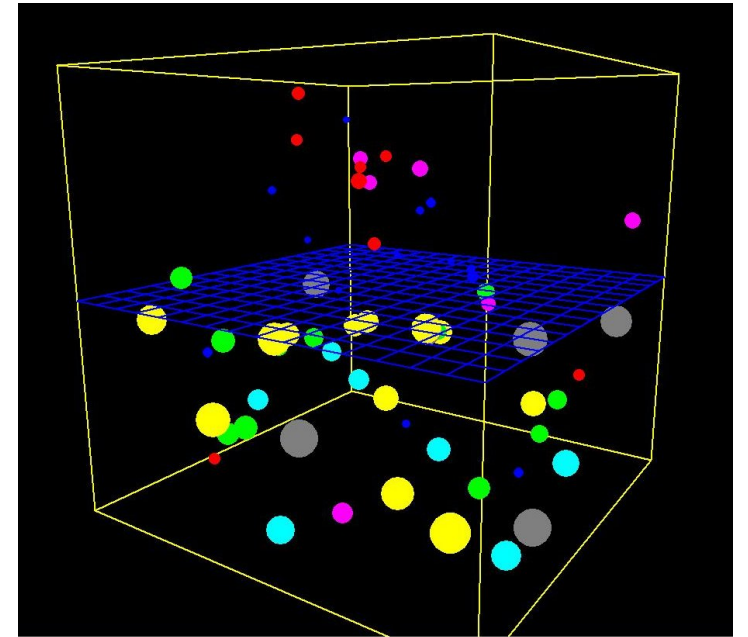
MATERIAL COVERED: LE01-05, HW01 Q17-42, HW02 Q1-3

Goals for Today

- Introduction to gases
 - Define and describe gases qualitatively
- Modeling Gases
 - Discuss the eponymous gas laws
 - Combine the gas laws into the ideal gas law
 - Extend the ideal gas law to quantify density
- Gas Law Stoichiometry
 - Add one layer of complexity to stoichiometry problems
- Maybe: Introduction to Kinetic Molecular Theory
 - Describe the assumptions of KMT
 - Introduce the relationships between kinetic energy, temperature
 - Introduce the relationships between velocity and mass of a gas

Introduction to Gases

- As an introduction to gases, we pretend that all gases are “ideal gases.”
- This comes with a few assumptions:
 - A gas consists of a bunch of molecules that obey Newtonian physics
 - Gas molecules are tiny spheres and they all have the same volume – however, this volume is so small we consider it negligible. **Later on, we will explain that this means there are no repulsive forces present.**
 - No forces act upon these gas molecules except for instantaneous, perfectly elastic collisions – **in particular, gas molecules do not attract each other**



Yes, we will refute these assumptions later.

Modeling Gases

- We describe ideal gases using **state functions**
 - State functions: a quantity or description that explains the current state of a chemical system
 - Another way to describe them is that they explain the final and initial, but not the process in between
- The state functions we use to describe gases are:
 1. Pressure (P): number of **collisions** between the gas and the walls of the container
 2. Volume (V): the volume of the **container**
 3. Temperature (T): **Kelvin**
 4. Number of moles (n): the **quantity** of gas present
- For now, we can use a few very important relationships to model gases.

Modeling Gases: Common Laws

Charles Law

- The volume and temperature of a gas are directly proportional
- As temperature increases, volume increases
- $V/T = k$
- $V_1/T_1 = V_2/T_2$

Boyle's Law

- The pressure and volume of a gas are inversely proportional
- As volume decreases, pressure increases
- $PV = k$
- $P_1V_1 = P_2V_2$

Avogadro's Law

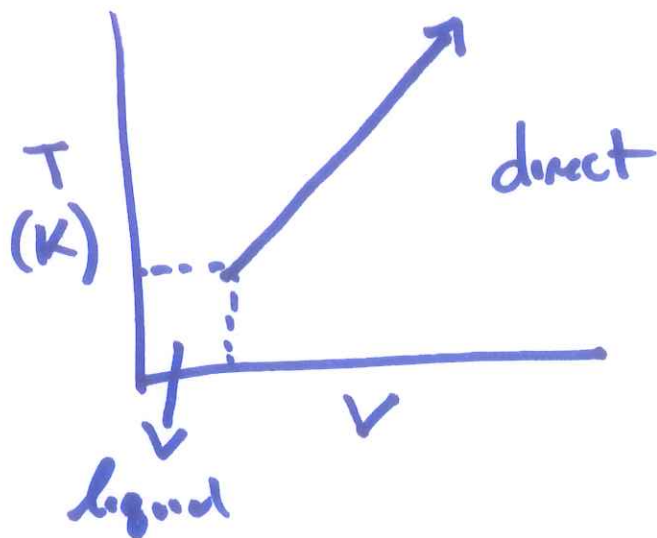
- The volume and number of moles of a gas are directly proportional
- $V/n = k$
- $V_1/n_1 = V_2/n_2$

- Amonton's Law relates Pressure and Temperature in the same way as Charles Law (no one cares about it for some reason)
- The "Bike Pump" Law relates Pressure and number of moles in a direct proportionality (pump in more moles of gas, the pressure goes up)

1) Charles' Law

$$\frac{V}{T} = \underline{K_{\text{constant}}}$$

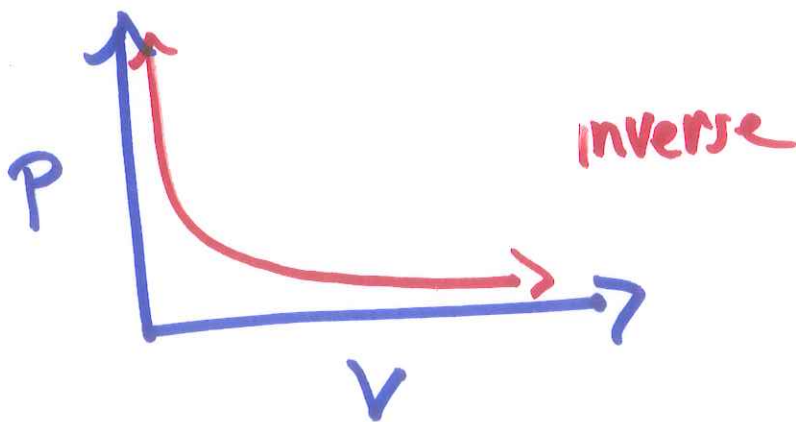
$\uparrow T, \uparrow V$



2) Boyle's Law

$$PV = K$$

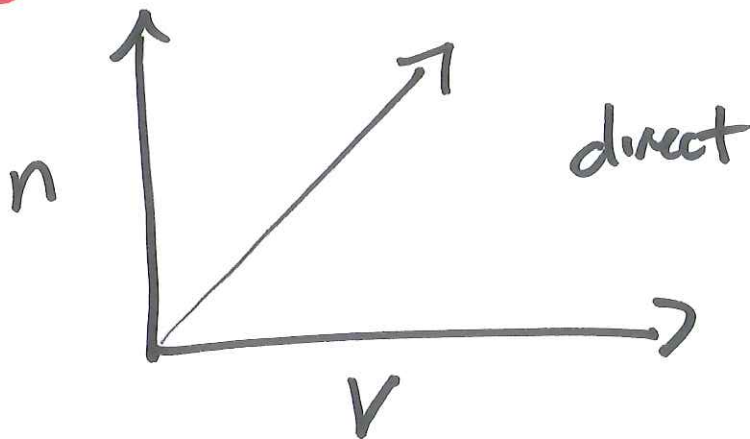
$\uparrow P, \downarrow V$



3) Avogadro's Law

$$\frac{V}{n} = K$$

$\uparrow n, \uparrow V$



Exam-Style Questions

If you have a gas in a closed, flexible container (like a balloon, for example) with a volume of 10L and a pressure of 0.5atm. What volume container is necessary to create a system with a pressure of 1atm with the same amount of gas?

Let's go back to our initial container: volume = 10L and pressure = 0.5atm. What is the volume of your container when you heat it from 25 degrees Celsius to 50 degrees Celsius at constant pressure?

Hint: for the second question, ask yourself first if it is as simple as doubling?

Exam-Style Questions

If you have a gas in a closed, flexible container (like a balloon, for example) with a volume of 10L and a pressure of 0.5atm. What volume container is necessary to create a system with a pressure of 1atm with the same amount of gas? **5L**

Let's go back to our initial container: volume = 10L and pressure = 0.5atm. What is the volume of your container when you heat it from 25 degrees Celsius to 50 degrees Celsius at constant pressure? **10.84 L**

Hint: for the second question, ask yourself first if it is as simple as doubling? **Very small change**

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}, \quad \frac{10L}{298K} = \frac{V_2}{323K}$$

$$\left(\frac{10L}{298K} \right) 323K = \underline{10.84L}$$

All Eponymous Gas Laws

Pretty much everything on the exam can be solved using the ideal gas law, but don't forget the individual relationships that make it possible.

- **Boyle's Law:** Pressure and volume have an inverse relationship

$$P \propto \frac{1}{V}$$

- **Charles' Law:** Volume and temperature have a direct relationship

$$V \propto T$$

- **Avogadro's Law:** Volume and number of moles have a direct relationship

$$V \propto n$$

- **Bike Pump Law:** Pressure and number of moles have a direct relationship

$$P \propto n$$

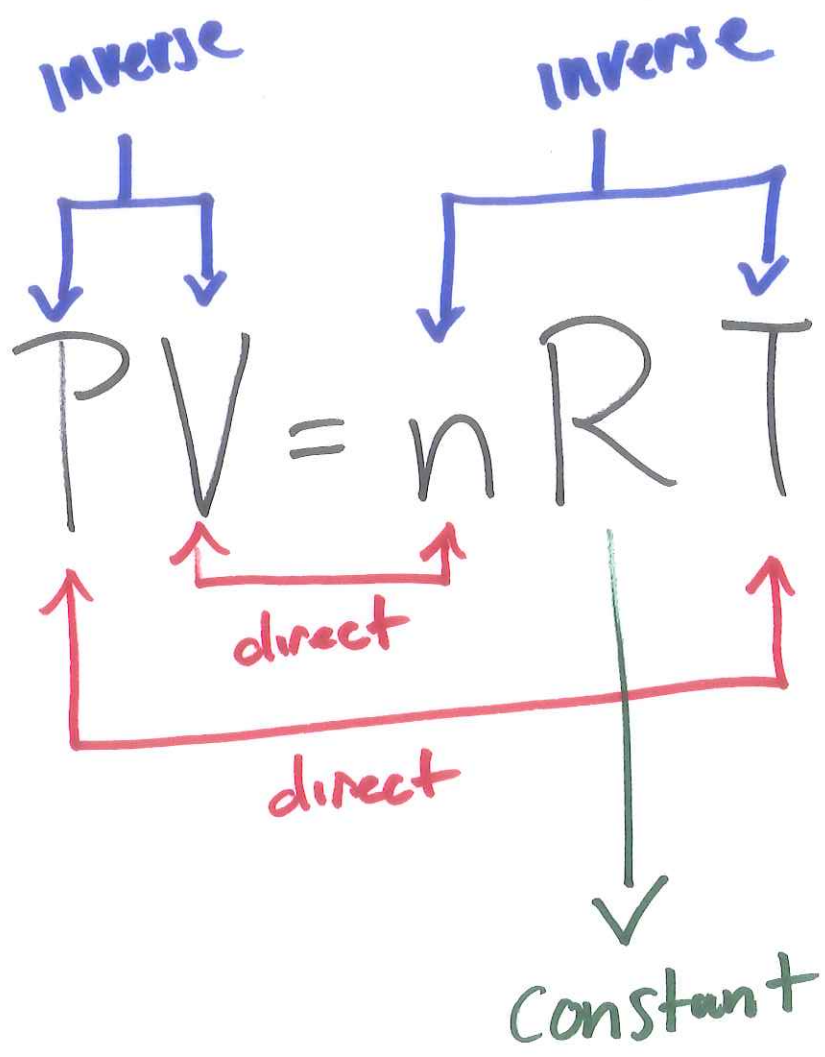
- **Amonton's Law:** Pressure and temperature have a direct relationship

$$P \propto T$$

- **Hard-to-do-this Law:** number of moles and temperature have an inverse relationship

$$n \propto \frac{1}{T}$$

- Or just remember that given $PV=nRT$:
 - State functions on the same side have an **inverse** relationship
 - State functions on opposite sides have a **direct** relationship



Modeling Gases: The Ideal Gas Law

- The relationship between these state functions is presented in the following equation:

$$PV = nRT$$

- How we use it: we will use this formula when we change one state function and keep all other state functions except one constant.
- Examples:
 - If you **double** pressure and keep the same number of moles and temperature, the volume will **half**.
 - If you **double** the temperature and maintain the same number of moles and pressure, the volume will **double**.
- What about R? R will never change value **unless you change the units** because it is a constant. Constants in the natural sciences are used to relate state functions with different units. *****Make sure you use the correct R value*****
 - **Note: R is NOT a state function**

The Universal Gas Constant - R

$$PV = nRT$$

- The reason why this relationship works as an equality is because of the universal gas constant, R
- The role of R is to cancel your units. If R does not match the units of pressure, volume, moles, and temperature, you will end up with the wrong value.
 - The units of R stem from the following relationship:

$$\frac{PV}{nT} = R$$

- For this exam, the R-value you choose will be based on your unit of **pressure** in the question

$$\frac{(atm)(L)}{(mol)(K)} = 0.08206 L \cdot atm \cdot mol^{-1} \cdot K^{-1}$$

$$\frac{(torr)(L)}{(mol)(K)} = 62.364 L \cdot torr \cdot mol^{-1} \cdot K^{-1}$$

Units of R:

$$\frac{L \cdot \text{Pressure Unit}}{\text{mol} \cdot K}$$

always L, mol, K

determines the value of R, depends on units of pressure in question

* exception: KMT will use $8.314 \frac{J}{\text{mol} \cdot K}$

↓
only use when dealing with energy

Quick Question

STP is short for “Standard temperature and pressure.” For gases this is defined as 0 degrees Celsius and 1atm pressure. Knowing this, what is the volume of 1 mole gas at STP?

$$R = 0.08206 \text{ L atm/mol K}$$

$$R = 8.314 \text{ J/mol K}$$

$$R = 62.364 \text{ L torr/mol K}$$

Note: The value you get for this answer is definitely worth memorizing for the exam.

$$PV = nRT$$

↓ ↓ ↓

1 atm = 1 273 K

0.08206 $\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}$

$$V = 22.4 \text{ L}$$

The Ideal Gas Law and Density

$$\rho = \frac{MW \cdot P}{R \cdot T} \quad \text{or} \quad MW = \frac{\rho \cdot R \cdot T}{P}$$

- You can solve for the mass density of a gas using the molecular weight (or vice versa) with a simple modification of the ideal gas law.
- What is density?
 - Number density = $\frac{n}{V}$
 - Mass density (or just density) = $\frac{m}{V} = \rho$
- Two take-away conceptual points:
 - **All other conditions being equal in the ideal gas law equation (P, V, & T), all ideal gases have the same number density; number density depends on P, V, & T but NOT molecular weight**
 - **All other conditions being equal in the ideal gas law equation (P, V, & T), a heavier gas will have a greater mass density; mass density depends on the molecular weight of the gas**

McCordian Question

You have 2.39g ideal gas that occupies 3L at 740 torr and 300K. Which ideal gas is it?

$$R = 0.08206 \text{ L atm/mol K}$$

$$R = 8.314 \text{ J/mol K}$$

$$R = 62.364 \text{ L torr/mol K}$$

- a. Helium (**4.00 g/mol**),
- b. Neon (**20.2 g/mol**),
- c. Argon (**39.95 g/mol**),
- d. Krypton (**83.8 g/mol**),
- e. Xenon (**131.3 g/mol**)

$$1) \frac{P \cancel{V}}{\cancel{V}} = \left(\frac{n}{\cancel{V}} \right) R T$$

cancel \rightarrow # density

$$2) MW \cdot P = \frac{n}{V} R T \cdot MW$$

$$3) MW \cdot P = \left(\frac{m}{V} \right) R T$$

\rightarrow mass density, ρ

$$MW = \frac{P R T}{P}$$

a) solve for
molecular
weight

$$P = \frac{MW \cdot P}{R T}$$

b) solve for mass
density

$$MW = \frac{\rho RT}{P}$$

ρ → mass density
 P → Pressure

Solve:

$$\frac{(0.8 \frac{g}{L}) (62.364 \frac{L \cdot torr}{mol \cdot K}) (300K)}{740 torr}$$

$$= 20.2 \frac{g}{mol}$$

$$T = 300K$$

$$P = 740 torr$$

$$R = 62.364$$

$$P = 0.974 atm$$

$$R = 0.08206$$

$$\rho = \frac{m}{V} = \frac{2.39g}{3L}$$

$$= 0.8 \frac{g}{L}$$

McCordian Question

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- a. Helium (4.00 g/mol),
- b. **Neon (20.2 g/mol),**
- c. Argon (39.95 g/mol),
- d. Krypton (83.8 g/mol),
- e. Xenon (131.3 g/mol)

Remember from last week...

We have covered the important fundamentals of stoichiometry. However, when studying you should always think: **how can we make these questions more fun** (a.k.a. harder)? Some examples include:

- **Convert your number of moles into mass or weight**

- Use dimensional analysis to convert from moles to grams to pounds, etc.
- Dr. B's class: know how to do this by tomorrow!

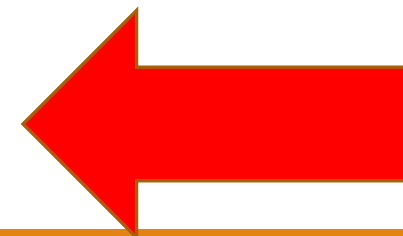
- **Account for an experimental percent yield**

- Simply multiply your final answer by the percent yield
- For example: in the last problem our answer was 2.50 moles of iron (III) oxide. This represents a 100% **theoretical yield**. If the actual percent yield is 50%, your **experimental yield** is 1.25 moles (2.50 x 0.50).

$$\frac{\text{Experimental Yield}}{\text{Theoretical Yield}} \cdot 100\% = \text{Percent Yield}$$

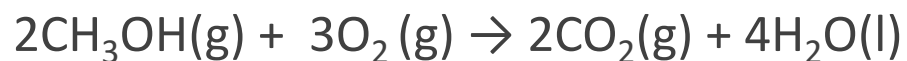
- **Add a gas law problem at the end (we will cover this later in Unit 1)**

- For example, use the total volume of gases to solve for the total pressure



Maximum Difficulty Stoichiometry Problem!!!

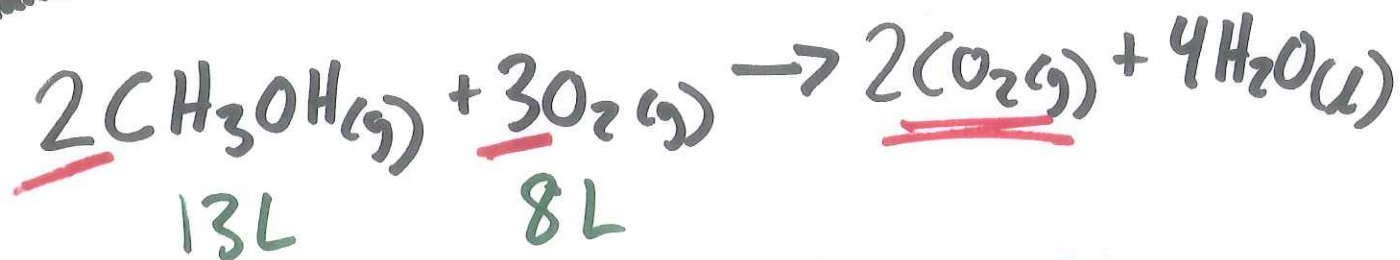
Consider the combustion of methanol at STP.



If 13L of methanol reacts with 8L oxygen, how many moles of gas are present in the final reaction mixture?

@STP, 22.4 L/mol

(i) Balance



(ii) Identify the L.R.

$$\text{CH}_3\text{OH} \quad 13\text{L} \times \frac{\text{reaction}}{2\text{L}} = 6.5 \text{ reactions}$$

$$\text{O}_2 \quad 8\text{L} \times \frac{\text{reaction}}{3\text{L}} = 2.67 \text{ reactions}$$

L.R.

(iii) How much products?

$$2.67 \text{ reactions} \times \frac{2\text{L CO}_2}{1 \text{ reaction}} = 5.33\text{L CO}_2$$

* H_2O is a liquid, has negligible volume*

(iv) How much excess?

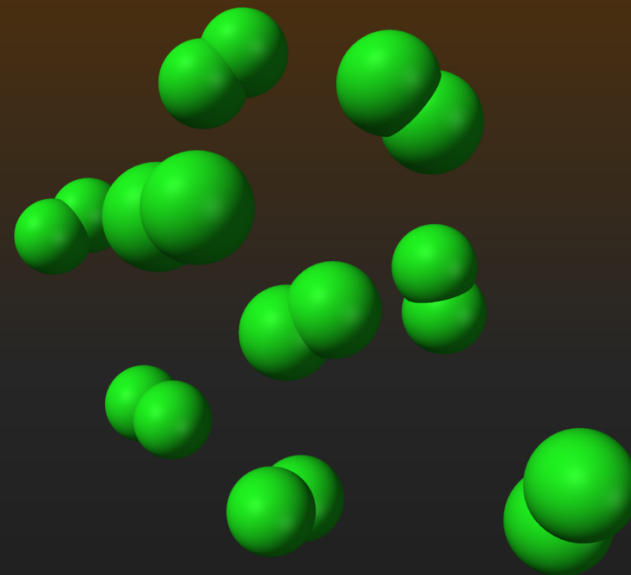
$$\text{excess} = \text{initial} - \text{amount USED}$$

$$7.66\text{L} = 13\text{L} - \left(2.67 \text{ rxn} \times \frac{2\text{L}}{\text{rxn}} \right)$$

$$(v) \text{ Sum: } 5.33 + 7.66 = 13\text{L}$$

$$PV = nRT \quad \text{OR} \quad 13\text{L} \times \frac{\text{mol}}{22.4\text{L}} = 0.58 \text{ moles}$$

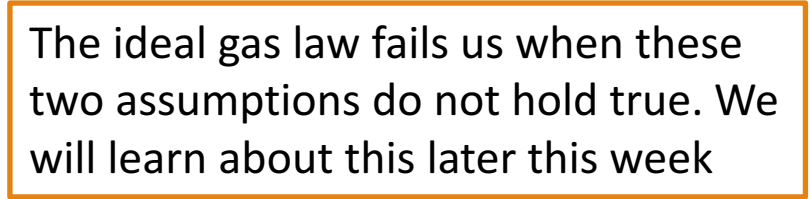
Kinetic Molecular Theory



Kinetic Molecular Theory

Kinetic Molecular Theory explains the “energetic” component of gases, including kinetic energy, velocity, and the relationship between kinetic energy and temperature

1. Gases are constantly moving in random directions
2. The distance between particles is large compared to the particle size
 - True ideal gases have relatively **no volume**
3. All particles have perfectly elastic collisions
 - There is no energy loss in the system to collisions; energy cannot be created or destroyed based on Newtonian Physics
4. No other forces act upon ideal gases
 - There are no attractive or repulsive forces that act upon ideal gas particles



The ideal gas law fails us when these two assumptions do not hold true. We will learn about this later this week

Main conclusions: the ideal gas law works because when these pillars of KMT hold true in a system. The ideal gas law is modeled best at **High Temperature** and **Low Pressure**

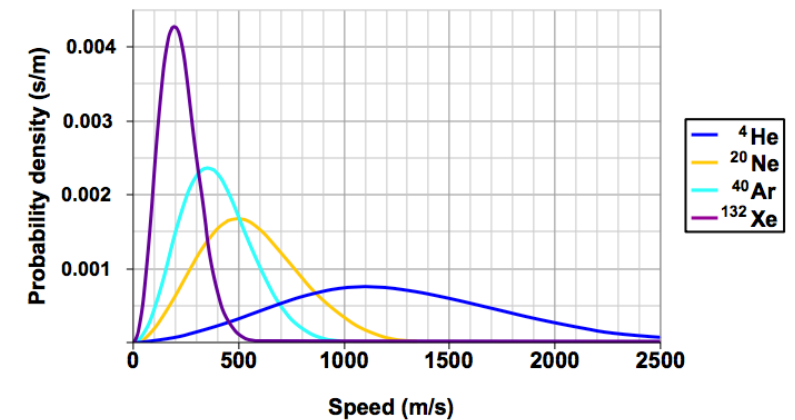
KMT: Conceptual Points

1. Kinetic energy depends **ONLY** on temperature

$$E_k = \frac{3}{2}RT$$

2. When you have different gases at a given temperature:
 - The heavy gases move slower on average
 - **The lighter gases move faster on average**
 - **All gases have the same kinetic energy**
3. When you have the same gas at different temperatures:
 - The samples have different kinetic energies because the temperatures is different and **kinetic energy depends on temperature**
 - **The hotter gas sample moves faster on average**
 - The colder gas sample moves slower on average
4. Gas velocity is given as a statistical average of velocities, v_{rms}
 - Gases move at a distribution of speeds, although **the average kinetic energy of the sample depends only on temperature**

Maxwell-Boltzmann Molecular Speed Distribution for Noble Gases



Summary:

- **Ideal gases are pretty easy if you use the ideal gas law properly.**
 - Students who get these problems wrong usually do one of two things:
 - Their R value is wrong
 - They do not convert from Celsius to Kelvin
- **Stoichiometry is important for gases, so learn it**
 - Remember to differentiate between questions asking for the quantity of product and the quantity of total species in the final system
- **The average kinetic energy of a system is only dependent on the temperature**
- Coming up:
 - The equations of KMT
 - Mixtures of gases: partial pressure and concentrations
 - Deviating from ideal gases: understanding that gas molecules have volume and there are very small but non-negligible interactions between them
 - Exam!

Kinetic Molecular Theory: Relationships

- Kinetic Molecules Theory gives us three key relationships that you should know as equations and by definition (in words)

1. Kinetic Energy vs. Temperature

$$E_k = \frac{3}{2}RT \leftarrow \text{Energy per mole (J/mol)}$$

- **Kinetic energy is dependent solely on the temperature of a gaseous system (direct relationship)**
- $R = 8.314 \text{ J / mol K}$

$$E_k = \frac{3}{2}kT \leftarrow \text{Energy per molecule (J)}$$

- $k = \frac{R}{N_a}$

2. Mass vs. Velocity

3. Temperature vs. Velocity

Remember that in physics we describe kinetic energy with the equation:

$$\frac{1}{2}mv^2$$

KMT: Mass vs. Velocity; Velocity vs. Temperature

1. Kinetic Energy vs. Temperature

Based on the equation:

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$R = 8.314 \text{ J/mol K}$

Molecular Weight in Kg

We can determine that velocity is proportional to the **square root of temperature** and the **inverse square root** of mass.

1. Mass vs. Velocity (V_{rms})

- Velocity is proportional to the inverse square root of mass.
- When temperature is constant, lighter particles move faster

$$\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}}$$

Pay very close attention to how the 1 and 2 match up in these relationships

2. Velocity (V_{rms}) vs. Temperature

- Velocity is proportional to the square of temperature
- When dealing with the same species gas, particles move faster at higher temperatures

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

Practice Problem

What is the ratio of the effusion rates of SO_2 to Cl_2 ?

$$\frac{v_{\text{SO}_2}}{v_{\text{Cl}_2}} = \sqrt{\frac{M_{\text{Cl}_2}}{M_{\text{SO}_2}}}$$

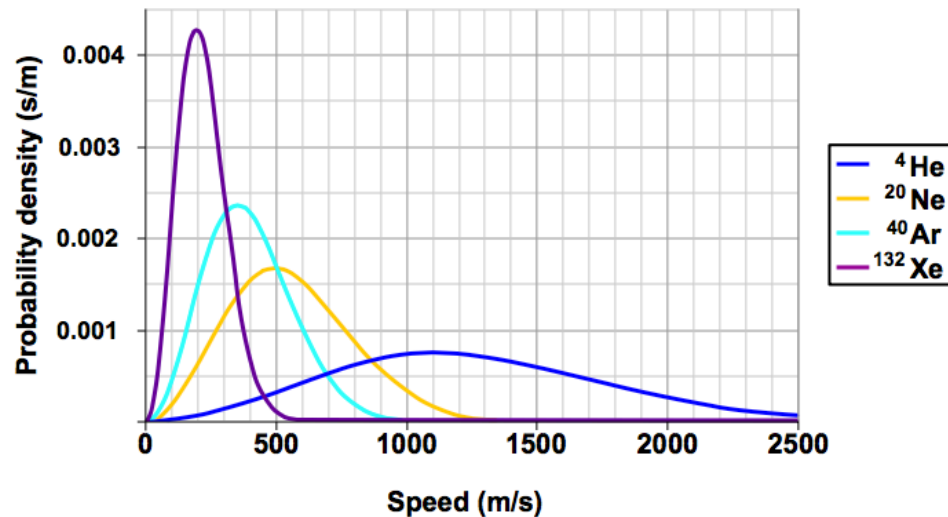
The position of these is important because it is asking you for the ratio of $\text{SO}_2 : \text{Cl}_2$

If the velocity of Cl_2 is 500m/s, what is the temperature?

$$V_{\text{SO}_2} = \sqrt{\frac{3RT}{M}}$$

KMT: Mass vs. Velocity; Velocity vs. Temperature

Maxwell-Boltzmann Molecular Speed Distribution for Noble Gases



- Some key features of this graph include:
 - Each curve looks like a unimodal distribution with a “tail” that approaches the limit infinite velocity (0 probability density)
 - Molecules are traveling at a variety of speeds but there is a clear average
 - The actual V_{rms} is slightly to the right of the peak

1. Mass vs. Velocity (V_{rms})

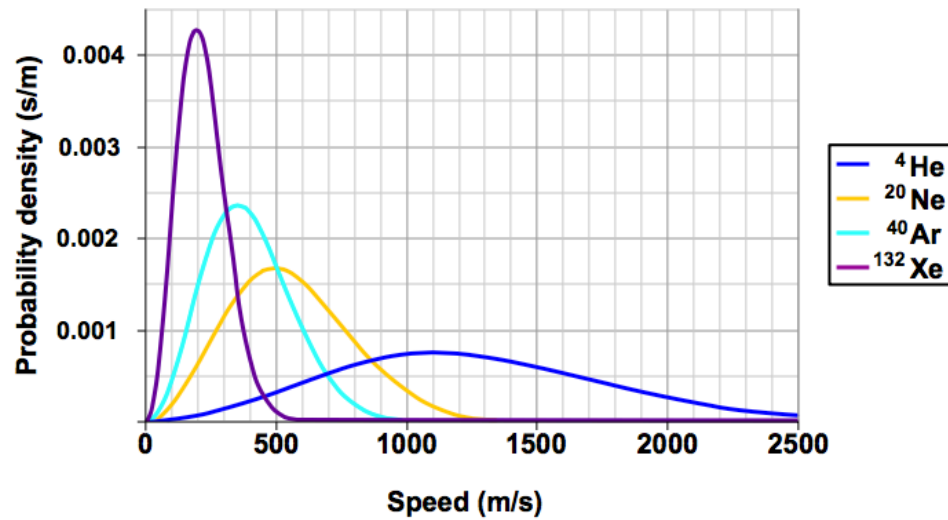
- We can see that the **heavier gases** move **slower** and the **lighter gases** move **faster**
- The **faster** the gas, the **wider** the distribution

2. Velocity (V_{rms}) vs. Temperature

- If you were working with the same gas, a similar graph could be created by modifying temperature instead.
- In this case, higher temperatures of the same gas result in faster speeds.

Maxwell-Boltzmann Distribution

Maxwell-Boltzmann Molecular Speed Distribution for Noble Gases



- Given a Maxwell distribution, you should know:
 - Find the approximate V_{rms}
 - Be able to label **which gas is which** (or which temperature is which)
 - Understand the **relationships** between mass, velocity, and temperature
 - Understand how these relationships impact the **shape** of the curve